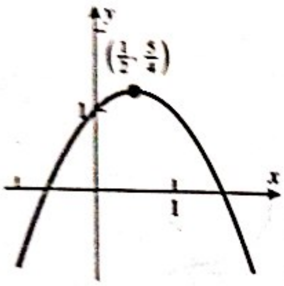
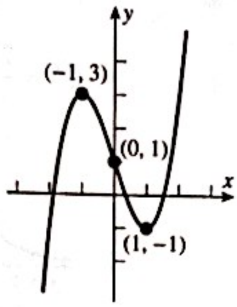
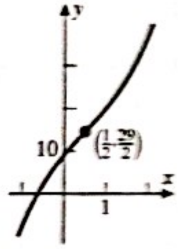
2. $y = 1 + x - x^2$;
$y' = -2(x - 1/2)$;
$y'' = -2$



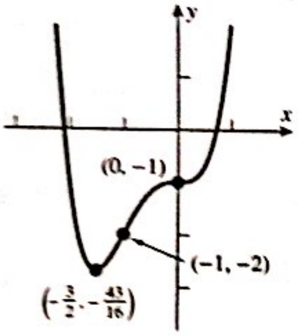
3. $y = x^3 - 3x + 1$;
$y' = 3(x^2 - 1)$;
$y'' = 6x$



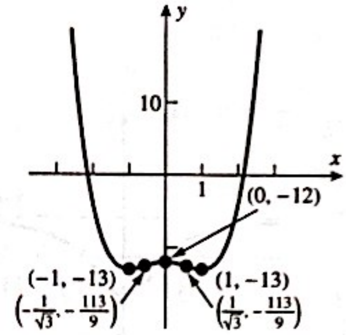
4. $y = 2x^3 - 3x^2 + 12x + 9$;
$y' = 6(x^2 - x + 2)$;
$y'' = 12(x - 1/2)$



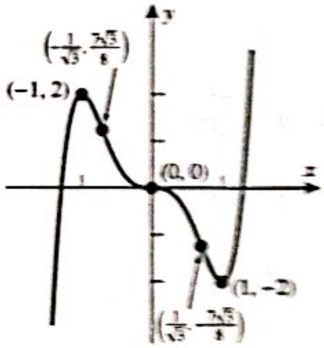
5. $y = x^4 + 2x^3 - 1$;
$y' = 4x^2(x + 3/2)$;
$y'' = 12x(x + 1)$



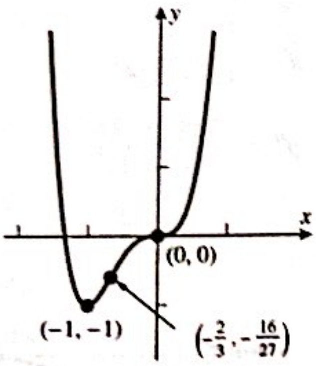
6. $y = x^4 - 2x^2 - 12$;
$y' = 4x(x^2 - 1)$;
$y'' = 12(x^2 - 1/3)$



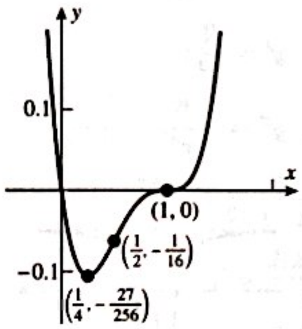
7. $y = x^3(3x^2 - 5)$;
$y' = 15x^2(x^2 - 1)$;
$y'' = 30x(2x^2 - 1)$



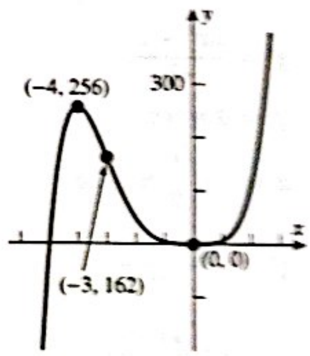
8. $y = 3x^3(x + 4/3)$;
$y' = 12x^2(x + 1)$;
$y'' = 36x(x + 2/3)$



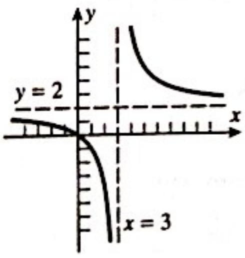
9. $y = x(x - 1)^3$;
$y' = (4x - 1)(x - 1)^2$;
$y'' = 6(2x - 1)(x - 1)$



10. $y = x^4(x + 5)$;
$y' = 5x^3(x + 4)$;
$y'' = 20x^2(x + 3)$



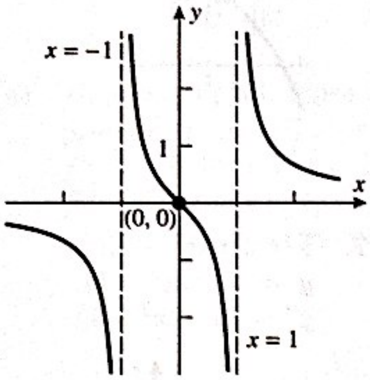
11. $y = 2x/(x - 3)$;
$y' = -6/(x - 3)^2$;
$y'' = 12/(x - 3)^3$



12. $y = \dfrac{x}{x^2 - 1};$

$y' = -\dfrac{x^2 + 1}{(x^2 - 1)^2};$

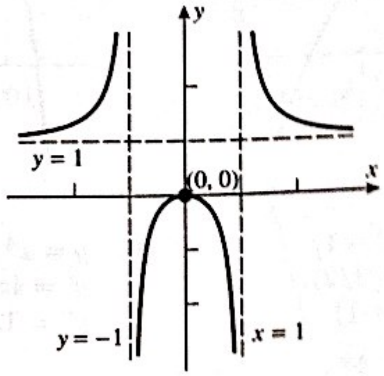
$y'' = \dfrac{2x(x^2 + 3)}{(x^2 - 1)^3}$



13. $y = \dfrac{x^2}{x^2 - 1};$

$y' = -\dfrac{2x}{(x^2 - 1)^2};$

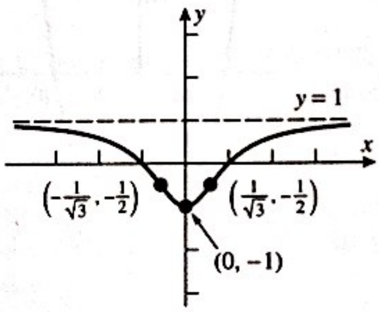
$y'' = \dfrac{2(3x^2 + 1)}{(x^2 - 1)^3}$



14. $y = \dfrac{x^2 - 1}{x^2 + 1};$

$y' = \dfrac{4x}{(x^2 + 1)^2};$

$y'' = \dfrac{4(1 - 3x^2)}{(x^2 + 1)^3}$



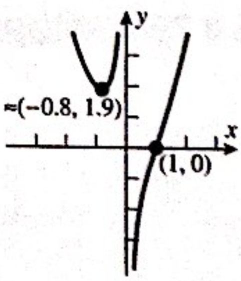
15. $y = x^2 - \dfrac{1}{x} = \dfrac{x^3 - 1}{x};$

$y' = \dfrac{2x^3 + 1}{x^2},$

$y' = 0$ when

$x = -\sqrt[3]{\dfrac{1}{2}} \approx -0.8;$

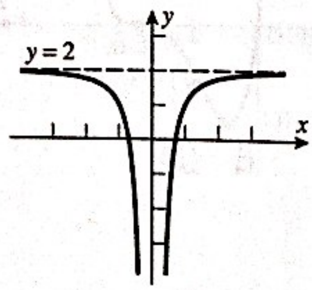
$y'' = \dfrac{2(x^3 - 1)}{x^3}$



16. $y = \dfrac{2x^2 - 1}{x^2};$

$y' = \dfrac{2}{x^3};$

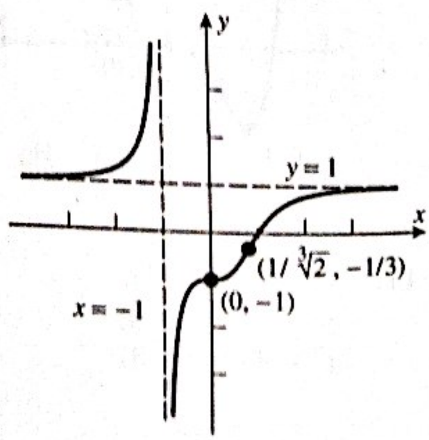
$y'' = -\dfrac{6}{x^4}$



17. $y = \dfrac{x^3 - 1}{x^3 + 1};$

$y' = \dfrac{6x^2}{(x^3 + 1)^2};$

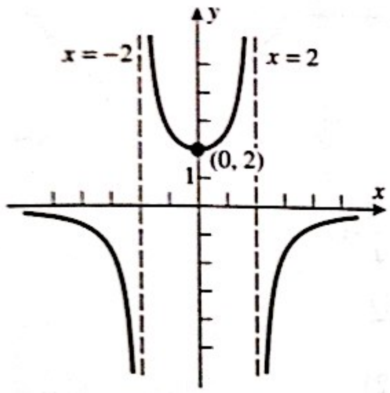
$y'' = \dfrac{12x(1 - 2x^3)}{(x^3 + 1)^3}$



18. $y = \dfrac{8}{4 - x^2}$;

$y' = \dfrac{16x}{(4 - x^2)^2}$;

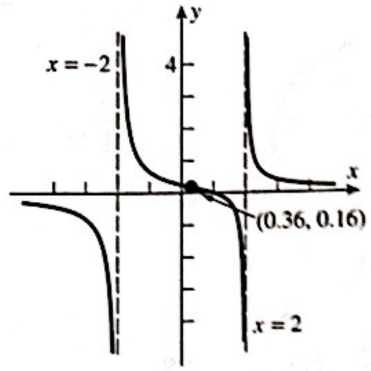
$y'' = \dfrac{16(3x^2 + 4)}{(4 - x^2)^3}$



19. $y = \dfrac{x - 1}{x^2 - 4}$;

$y' = -\dfrac{x^2 - 2x + 4}{(x^2 - 4)^2}$

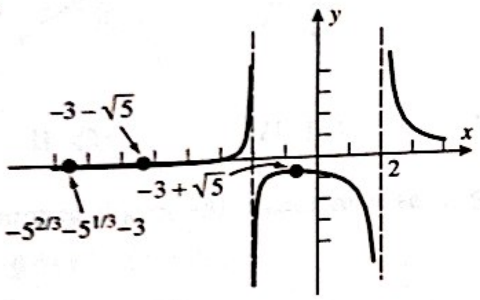
$y'' = 2\dfrac{x^3 - 3x^2 + 12x - 4}{(x^2 - 4)^3}$



20. $y = \dfrac{x + 3}{x^2 - 4}$;

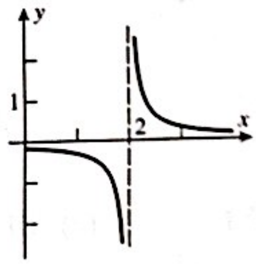
$y' = -\dfrac{(x^2 + 6x + 4)}{(x^2 - 4)^2}$

$y'' = 2\dfrac{x^3 + 9x^2 + 12x + 12}{(x^2 - 4)^3}$



21. $y = \dfrac{1}{x - 2}$;

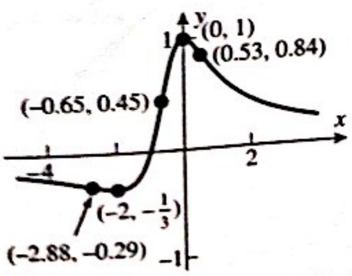
$y' = \dfrac{-1}{(x - 2)^2}$



22. $y = \dfrac{x^2 - 1}{x^3 - 1} = \dfrac{x + 1}{x^2 + x + 1}$;

$y' = -\dfrac{x(x + 2)}{(x^2 + x + 1)^2}$

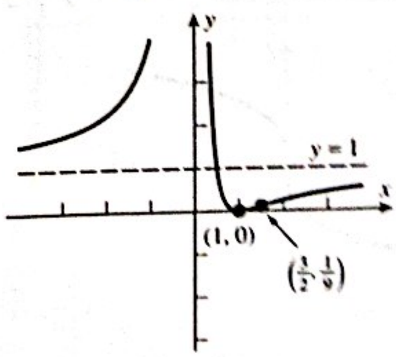
$y'' = 2\dfrac{x^3 + 3x^2 - 1}{(x^2 + x + 1)^3}$



23. $y = \dfrac{(x - 1)^2}{x^2}$;

$y' = \dfrac{2(x - 1)}{x^3}$;

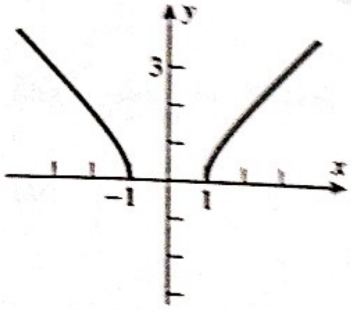
$y'' = \dfrac{2(3 - 2x)}{x^4}$



29. $y = \sqrt{x^2 - 1}$;

$y' = \dfrac{x}{\sqrt{x^2 - 1}}$;

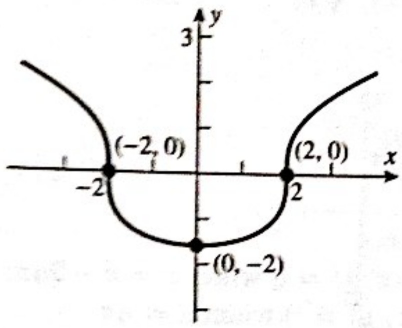
$y'' = -\dfrac{1}{(x^2 - 1)^{3/2}}$



30. $y = \sqrt[3]{x^2 - 4}$;

$y' = \dfrac{2x}{3(x^2 - 4)^{2/3}}$;

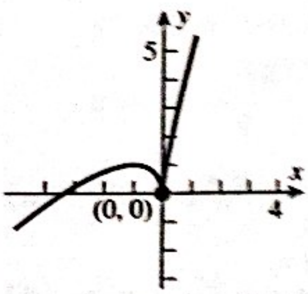
$y'' = -\dfrac{2(3x^2 + 4)}{9(x^2 - 4)^{5/3}}$



31. $y = 2x + 3x^{2/3}$;

$y' = 2 + 2x^{-1/3}$;

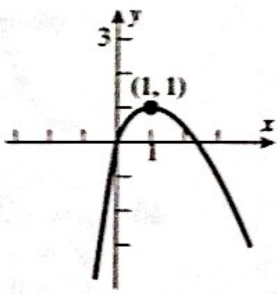
$y'' = -\dfrac{2}{3}x^{-4/3}$



32. $y = 4x - 3x^{4/3}$;

$y' = 4 - 4x^{1/3}$;

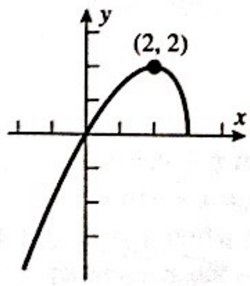
$y'' = -\dfrac{4}{3}x^{-2/3}$



33. $y = x(3 - x)^{1/2}$;

$y' = \dfrac{3(2 - x)}{2\sqrt{3 - x}}$;

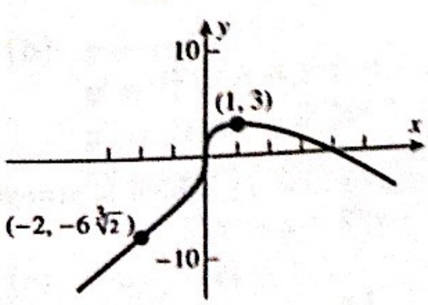
$y'' = \dfrac{3(x - 4)}{4(3 - x)^{3/2}}$



34. $y = x^{1/3}(4 - x)$;

$y' = \dfrac{4(1 - x)}{3x^{2/3}}$;

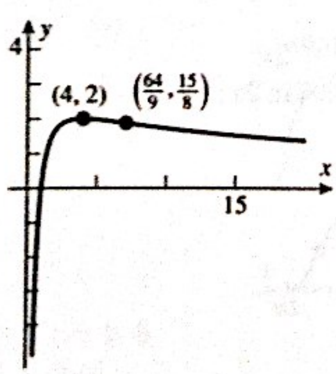
$y'' = -\dfrac{4(x + 2)}{9x^{5/3}}$



35. $y = \dfrac{8(\sqrt{x} - 1)}{x}$;

$y' = \dfrac{4(2 - \sqrt{x})}{x^2}$;

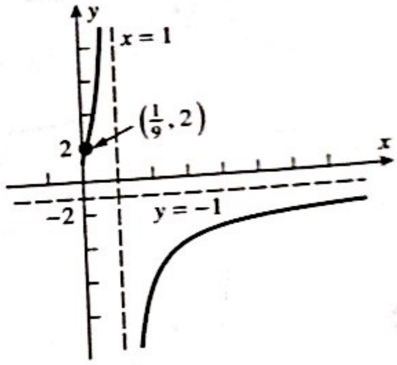
$y'' = \dfrac{2(3\sqrt{x} - 8)}{x^3}$



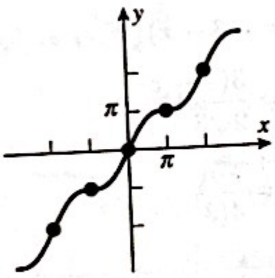
36. $y = \dfrac{1+\sqrt{x}}{1-\sqrt{x}}$;

$y' = \dfrac{1}{2\sqrt{x}(1-\sqrt{x})}$;

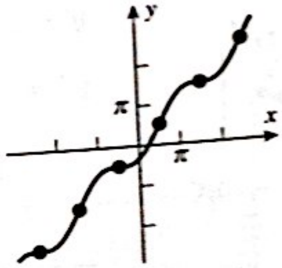
$y'' = \dfrac{3\sqrt{x}-1}{2x^{3/2}(1-\sqrt{x})^3}$



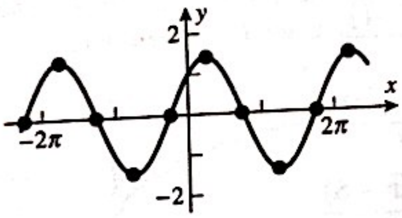
37. $y = x + \sin x$;
$y' = 1 + \cos x$, $y' = 0$ when $x = \pi + 2n\pi$;
$y'' = -\sin x$; $y'' = 0$ when $x = n\pi$
$n = 0, \pm 1, \pm 2, \ldots$



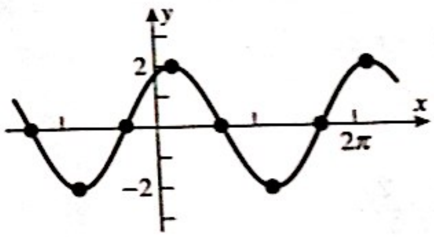
38. $y = x - \cos x$;
$y' = 1 + \sin x$;
$y' = 0$ when $x = -\pi/2 + 2n\pi$;
$y'' = \cos x$;
$y'' = 0$ when $x = \pi/2 + n\pi$
$n = 0, \pm 1, \pm 2, \ldots$



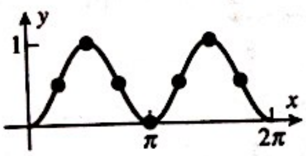
39. $y = \sin x + \cos x$;
$y' = \cos x - \sin x$;
$y' = 0$ when $x = \pi/4 + n\pi$;
$y'' = -\sin x - \cos x$;
$y'' = 0$ when $x = 3\pi/4 + n\pi$



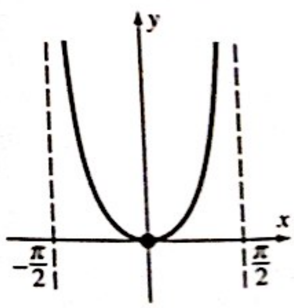
40. $y = \sqrt{3}\cos x + \sin x$;
$y' = -\sqrt{3}\sin x + \cos x$;
$y' = 0$ when $x = \pi/6 + n\pi$;
$y'' = -\sqrt{3}\cos x - \sin x$;
$y'' = 0$ when $x = 2\pi/3 + n\pi$



41. $y = \sin^2 x$, $0 \le x \le 2\pi$;
$y' = 2\sin x \cos x = \sin 2x$;
$y'' = 2\cos 2x$

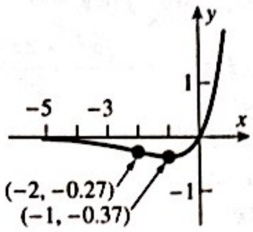


42. $y = x\tan x$, $-\pi/2 < x < \pi/2$;
$y' = x\sec^2 x + \tan x$;
$y' = 0$ when $x = 0$;
$y'' = 2\sec^2 x(x\tan x + 1)$, which is always positive for $-\pi/2 < x < \pi/2$



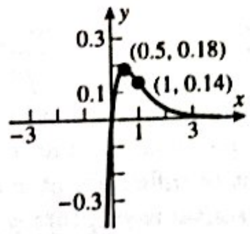
43. (a) $\lim_{x\to+\infty} xe^x = +\infty$, $\lim_{x\to-\infty} xe^x = 0$

(b) $y = xe^x$;
$y' = (x+1)e^x$;
$y'' = (x+2)e^x$



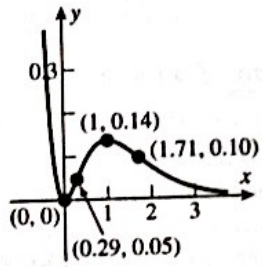
44. (a) $\lim_{x\to+\infty} xe^{-2x} = 0$, $\lim_{x\to-\infty} xe^{-2x} = -\infty$

(b) $y = xe^{-2x}$; $y' = -2\left(x - \frac{1}{2}\right)e^{-2x}$; $y'' = 4(x-1)e^{-2x}$



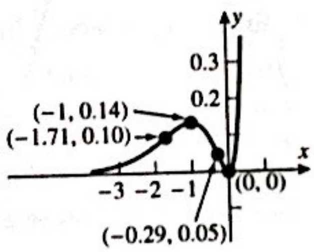
45. (a) $\lim_{x\to+\infty} \frac{x^2}{e^{2x}} = 0$, $\lim_{x\to-\infty} \frac{x^2}{e^{2x}} = +\infty$

(b) $y = x^2/e^{2x} = x^2e^{-2x}$;
$y' = 2x(1-x)e^{-2x}$;
$y'' = 2(2x^2 - 4x + 1)e^{-2x}$;
$y'' = 0$ if $2x^2 - 4x + 1 = 0$, when
$x = \frac{4 \pm \sqrt{16-8}}{4} = 1 \pm \sqrt{2}/2 \approx 0.29, 1.71$



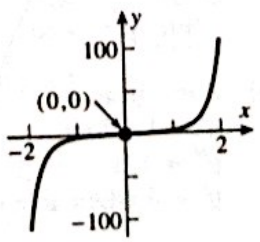
46. (a) $\lim_{x\to+\infty} x^2e^{2x} = +\infty$, $\lim_{x\to-\infty} x^2e^{2x} = 0$.

(b) $y = x^2e^{2x}$;
$y' = 2x(x+1)e^{2x}$;
$y'' = 2(2x^2 + 4x + 1)e^{2x}$;
$y'' = 0$ if $2x^2 + 4x + 1 = 0$, when
$x = \frac{-4 \pm \sqrt{16-8}}{4} = -1 \pm \sqrt{2}/2 \approx -0.29, -1.71$



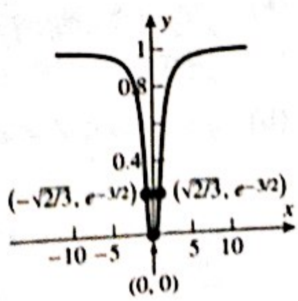
47. (a) $\lim_{x\to+\infty} f(x) = +\infty$, $\lim_{x\to-\infty} f(x) = -\infty$

(b) $y = xe^{x^2}$;
$y' = (1 + 2x^2)e^{x^2}$;
$y'' = 2x(3 + 2x^2)e^{x^2}$
no relative extrema, inflection point at $(0,0)$

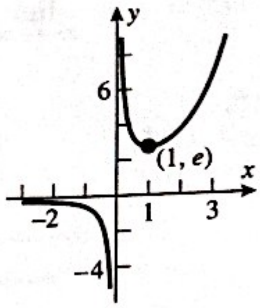


48. (a) $\lim_{x\to\pm\infty} f(x) = 1$

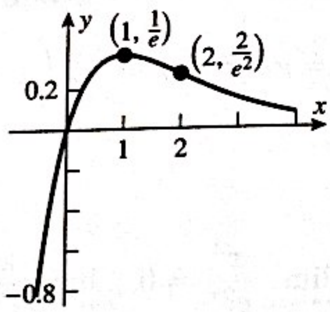
(b) $f'(x) = 2x^{-3}e^{-1/x^2}$ so $f'(x) < 0$ for $x < 0$ and $f'(x) > 0$ for $x > 0$. Set $u = x^2$ and use the given result to find $\lim_{x\to 0} f'(x) = 0$, so (by the First Derivative Test) $f(x)$ has a minimum at $x = 0$. $f''(x) = (-6x^{-4} + 4x^{-6})e^{-1/x^2}$, so $f(x)$ has points of inflection at $x = \pm\sqrt{2/3}$.



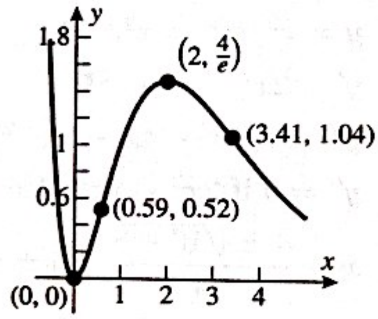
49. $\lim_{x\to+\infty} f(x) = +\infty,\ \lim_{x\to-\infty} f(x) = 0$
$f'(x) = e^x \dfrac{x-1}{x^2},\ f''(x) = e^x \dfrac{x^2-2x+2}{x^3}$
critical point at $x = 1$;
relative minimum at $x = 1$
no points of inflection
vertical asymptote $x = 0$,
horizontal asymptote $y = 0$ for $x \to -\infty$



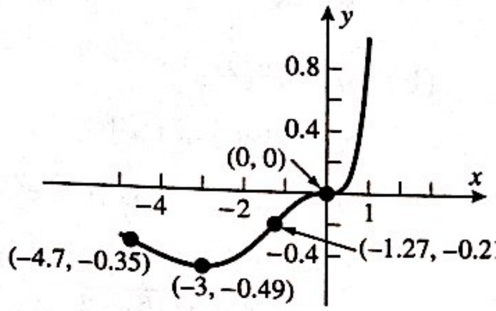
50. $\lim_{x\to+\infty} f(x) = 0,\ \lim_{x\to-\infty} f(x) = -\infty$
$f'(x) = (1-x)e^{-x},\ f''(x) = (x-2)e^{-x}$
critical point at $x = 1$;
relative maximum at $x = 1$
point of inflection at $x = 2$
horizontal asymptote $y = 0$ as $x \to +\infty$



51. $\lim_{x\to+\infty} f(x) = 0,\ \lim_{x\to-\infty} f(x) = +\infty$
$f'(x) = x(2-x)e^{1-x},\ f''(x) = (x^2-4x+2)e^{1-x}$
critical points at $x = 0, 2$;
relative minimum at $x = 0$,
relative maximum at $x = 2$
points of inflection at $x = 2 \pm \sqrt{2}$
horizontal asymptote $y = 0$ as $x \to +\infty$

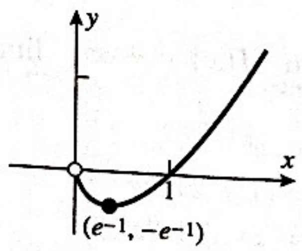


52. $\lim_{x\to+\infty} f(x) = +\infty,\ \lim_{x\to-\infty} f(x) = 0$
$f'(x) = x^2(3+x)e^{x-1},\ f''(x) = x(x^2+6x+6)e^{x-1}$
critical points at $x = -3, 0$;
relative minimum at $x = -3$
points of inflection at $x = 0, -3 \pm \sqrt{3} \approx 0, -4.7, -1.27$
horizontal asymptote $y = 0$ as $x \to -\infty$



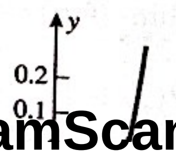
53. (a) $\lim_{x\to0^+} y = \lim_{x\to0^+} x\ln x = \lim_{x\to0^+} \dfrac{\ln x}{1/x} = \lim_{x\to0^+} \dfrac{1/x}{-1/x^2} = 0$;
$\lim_{x\to+\infty} y = +\infty$

(b) $y = x\ln x$,
$y' = 1 + \ln x$,
$y'' = 1/x$,
$y' = 0$ when $x = e^{-1}$



54. (a) $\lim_{x\to0^+} y = \lim_{x\to0^+} \dfrac{\ln x}{1/x^2} = \lim_{x\to0^+} \dfrac{1/x}{-2/x^3} = 0$,
$\lim_{x\to+\infty} y = +\infty$

(b) $y = x^2\ln x,\ y' = x(1+2\ln x)$,
$y'' = 3 + 2\ln x$,



55. (a) $\lim_{x\to 0^+} y = \lim_{x\to 0^+} \frac{\ln x}{x^2} = -\infty$;

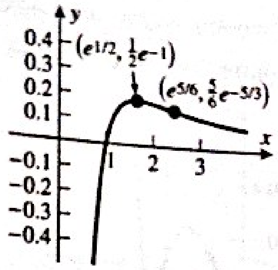
$\lim_{x\to +\infty} y = \lim_{x\to +\infty} \frac{\ln x}{x^2} = \lim_{x\to +\infty} \frac{1/x}{2x} = 0$

(b) $y = \frac{\ln x}{x^2}$, $y' = \frac{1 - 2\ln x}{x^3}$,

$y'' = \frac{6\ln x - 5}{x^4}$,

$y' = 0$ if $x = e^{1/2}$,

$y'' = 0$ if $x = e^{5/6}$



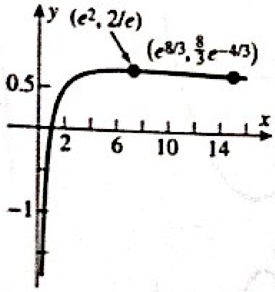
56. (a) Let $u = 1/x$, $\lim_{x\to 0^+} (\ln x)/\sqrt{x} = \lim_{u\to +\infty} -\sqrt{u}\ln u = -\infty$ by inspection, $\lim_{x\to +\infty} (\ln x)/\sqrt{x} = 0$, by the rule given.

(b) $y = \frac{\ln x}{\sqrt{x}}$, $y' = \frac{2 - \ln x}{2x^{3/2}}$

$y'' = \frac{-8 + 3\ln x}{4x^{5/2}}$

$y' = 0$ if $x = e^2$,

$y'' = 0$ if $x = e^{8/3}$



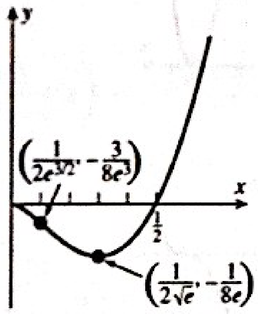
57. (a) $\lim_{x\to 0^+} x^2 \ln x = 0$ by the rule given, $\lim_{x\to +\infty} x^2 \ln x = +\infty$ by inspection, and $f(x)$ not defined for $x < 0$

(b) $y = x^2 \ln 2x$, $y' = 2x \ln 2x + x$

$y'' = 2\ln 2x + 3$

$y' = 0$ if $x = 1/(2\sqrt{e})$,

$y'' = 0$ if $x = 1/(2e^{3/2})$



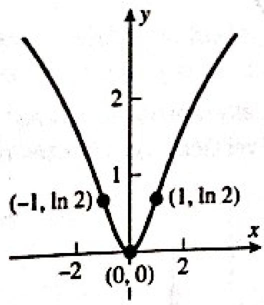
58. (a) $\lim_{x\to +\infty} f(x) = +\infty$; $\lim_{x\to 0} f(x) = 0$

(b) $y = \ln(x^2 + 1)$, $y' = 2x/(x^2 + 1)$

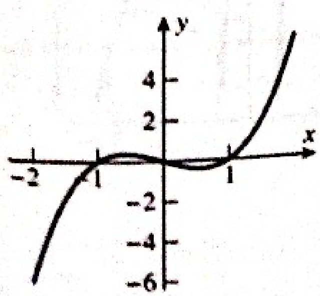
$y'' = -2\frac{x^2 - 1}{(x^2 + 1)^2}$

$y' = 0$ if $x = 0$

$y'' = 0$ if $x = \pm 1$



59. (a) $\lim_{x\to -\infty} y = -\infty$, $\lim_{x\to +\infty} y = +\infty$;
curve crosses x-axis at $x = 0, 1, -1$



(b) $\lim_{x\to \pm\infty} y = +\infty$;
curve never crosses x-axis